The distance to galactic globular clusters through RR Lyrae pulsational properties.

Cassisi S., De Santis R. & Piersimoni A.M.

 $Osservatorio\ Astronomico\ di\ Collurania,\ Via\ M.\ Maggini,\ I-64100,\ Teramo,\ Italy\ -\ E-Mail: cassisi, desantis, piersimoni@astrte.te.astro.it$

ABSTRACT

By adopting the same approach outlined in De Santis & Cassisi (1999), we evaluate the absolute bolometric magnitude of the Zero Age Horizontal Branch (ZAHB) at the level of the RR Lyrae variable instability strip in selected galactic globular clusters. This allows us to estimate the ZAHB absolute visual magnitude for these clusters and to investigate its dependence on the cluster metallicity. The derived $M_V(ZAHB) - [Fe/H]$ relation, corrected in order to account for the luminosity difference between the ZAHB and the mean RR Lyrae magnitude, has been compared with some of the most recent empirical determinations in this field, as the one provided by Baade-Wesselink analyses, RR Lyrae periods, Hipparcos data for field variables and Main Sequence fitting based on Hipparcos parallaxes for field subdwarfs. As a result, our relation provides a clear support to the "long" distance scale. We discuss also another method for measuring the distance to galactic globular clusters. This method is quite similar to the one adopted for estimating the absolute bolometric magnitude of the ZAHB but it relies only on the pulsational properties of the Lyrae variables in each cluster. The reliability and accuracy of this method has been tested by applying it to a sample of globulars for which, due to the morphology of their horizontal branch (HB), the use of the commonly adopted ZAHB fitting is a risky procedure. We notice that the two approaches, for deriving the cluster distance modulus, provide consistent results when applied to globular clusters whose RR Lyrae instability strip is well populated. Since the adopted method relies on theoretical predictions on both the fundamental pulsational equation and the allowed mass range for fundamental pulsators, we give an estimate of the error affecting present results, due to systematic uncertainties in the adopted theoretical framework.

Key words: stars: distances – stars: evolution – stars: horizontal branch – stars: variables: other – globular clusters: general

1 INTRODUCTION

RR Lyrae stars are the crossroad of several unsolved astrophysical problems. In fact, even though thorough observational and theoretical investigations have been devoted to analize the properties of these variables, we still lack a firm understanding of systematics affecting both the slope and the zero-point affecting the $M_V(RR)$ vs. [Fe/H] relation (Caputo 1997; Gratton 1998). This unpleasant fact raised the so called "distance dichotomy" between the "short" and the "long" distance scale. In particular, we are dealing with the empirical evidence that both the statistical parallaxes, and the Baade-Wesselink method of field RR Lyrae stars seem to support the "short" distance scale, whereas the pulsation properties of cluster RR Lyrae variables (Sandage 1993), the cluster main-sequence fitting to local subdwarfs (Gratton et al. 1997), and the calibration of HB luminosity based on the Large Magellanic Cloud distance obtained by adopting the Cepheid Period-Luminosity relation seem to support the "long" distance scale (Walker 1992). On the other hand, distance determinations obtained by adopting both evolutionary and pulsation predictions attain intermediate values between the "short" and the "long" distance scale. In addition, it has been suggested both by theoretical and empirical evidences that the RR Lyrae Luminosity-Metallicity relation - $M_V vs[Fe/H]$ - is not linear when moving from metal-poor to metal-rich RR Lyrae stars (for a careful review on all these topics we address to the reviews of Layden 1999, Popowski & Gould 1999 and Gratton, Carretta & Clementini 1999).

Up to now, a reliable estimate of the RR Lyrae luminosity - metallicity relation has been hampered by several problems, mainly related to the difficulties in estimating the individual cluster distances, reddening and metallicity of both cluster and field RR Lyrae variables.

This notwithstanding, the determination of the correct distance scale for Population II stellar systems has a large impact on a wide range of astrophysical problems, including the evaluation of the globular cluster (GC) ages which provides a stringent constraint on the lower limit to the age of the Universe, and the extra-galactic distances measurement which is a fundamental step in deriving the Hubble constant. Therefore, a precise determination of the absolute magnitude of the RR Lyrae variables - the traditional

distance ladder for metal poor stellar systems -, and of its dependence on the metallicity, is of crucial importance.

In view of the remarkably different results obtained so far with different methods, it is important to analyze the problem by using as many as possible, different and independent approaches in order to assess on more firm grounds the $M_V - \lceil Fe/H \rceil$ relation for RR Lyrae stars.

In a previous paper (De Santis & Cassisi 1999, hereinafter DC99), we have adopted an approach based on the pulsational behavior of RR Lyrae stars for obtaining an accurate estimate of the absolute bolometric luminosity of ZAHB stars in GCs.

We notice that the results obtained by DC99 relied on the ranking of HB stellar mass as a function of the effective temperature predicted by evolutionary models. It is worth noticing that the effective temperature of HB models, at variance with ZAHB luminosity, does not depend significantly on the physical inputs adopted in the stellar computations. As a consequence, it represents a rather reliable and trustworthy prediction of evolution theory. This occurrence allowed DC99 to perform a significant comparison with the most recent theoretical evaluations of the ZAHB luminosity (see Figure 15 in DC99).

In section 2 of this paper, we plan to use the results obtained in DC99, in order to derive an accurate estimation of both the slope and the zero point of the $M_V - [Fe/H]$ relation for ZAHB stellar structures. In the same section, after applying a correction for the difference between the mean RR Lyrae magnitude and the ZAHB one, we check the consistence between our $< M_V > (RRLyrae) - [Fe/H]$ relation and the most recent empirical ones. In section 3, we outline a method to determine the GCs distance based on the pulsational properties of their RR Lyrae population. The advantage of this method is that it does not need an estimate of the ZAHB level: a step, which is particularly risky in the case of GCs with blue HB, for which the RR Lyrae are suspected to be evolved stars. A brief discussion and conclusions follow in the last section.

2 THE $M_V(ZAHB)$ - [FE/H] RELATION.

The method employed for deriving the absolute bolometric luminosity of ZAHB structure at a fixed effective temperature (namely $\log T_e = 3.85$) inside the RR Lyrae instability strip, as well as the adopted temperature scale for the variables, have been extensively discussed by DC99. Therefore, a detailed discussion of the method will not be repeated here, and we address the reader to the quoted paper for more details.

However, we wish to briefly recall the fundamental steps of this approach. We make use of the results provided by the updated pulsational models by Bono et al. (1997), about the relationship among the fundamental period of the variable, its mass, luminosity and effective temperature. After rewriting the relation for the period as a function of evolutionary properties of the variable in order to obtain the dependence of the pulsational reduced period (P_{red}) (Sandage 1981) on the mass and effective temperature of the variable, and the absolute bolometric luminosity of the ZAHB at $\log T_e = 3.85$ ($L_{3.85}^{ZAHB}$), we have compared in the P_{red} - T_{eff} diagram the observational data for RR Lyrae stars with the theoretical prescriptions.

Figure 1. Panel a: The absolute visual ZAHB magnitude versus the iron content for the GCs in our sample, but NGC6171. The solid line represent our best fit to the observational points obtained when excluding the data for NGC6171 (see text for more details). Panel b: Comparison between different solutions for the mean absolute visual magnitude of RR Lyrae stars - metallicity relation as provided by various authors (see labels).

Since the theoretical relation for the reduced period depends on the mass of the variable, we have used as lower and upper limit on this mass the values provided by the evolutionary theory on the minimum and maximum stellar mass which can produce a RR Lyrae star at a given fixed metallicity.

Our approach to estimate the stellar masses of RR Lyrae stars relies on predictions of nonlinear pulsational models, namely the edges of the instability strip as well as on evolutionary predictions concerning the ranking of stellar masses as a function of the effective temperature. As a consequence, it is worth discussing whether adopted pulsational and evolutionary predictions supply consistent evaluations for the RR Lyrae masses. Fortunately enough, Bono et al. (1996) in a recent investigation by adopting the same pulsational scenario we are adopting, showed that pulsational and evolutionary masses for RR Lyrae stars are in fair agreement. Since, the evolutionary predictions (Castellani Chieffi & Pulone 1991), adopted by Bono et al. (1996) to construct the pulsational models are quite similar to the evolutionary framework adopted in this investigation, namely the ranking of the stellar masses with effective temperatures, we are confident that pulsational and evolutionary predictions adopted in the current analysis are internally consistent. The reader interested in a more quantitative discussion concerning the difference of RR Lyrae masses provided by different evolutionary HB models is referred to DC99.

Then, for each cluster it has been quite easy to determine the most suitable value for the intrinsic luminosity of the ZAHB structures by properly fitting the lower and upper boundaries of the RR Lyrae distribution in the reduced period - temperature plane. It is worth remembering that this

method allows us to estimate the bolometric ZAHB luminosity with high accuracy, and indeed the formal uncertainty on $\log L_{3.85}^{ZAHB}$ is of the order of ± 0.02 (for an accurate analysis of all uncertainties affecting the $L_{3.85}^{ZAHB}$ measurements we refer to DC99).

For clusters with also RRc (first overtone) variables a similar approach has been adopted, obtaining results in fine agreement with the ones derived from RRab stars.

In order to successfully apply this method, one needs clusters with homogeneous photometry for both variable and non-variable HB stars, and spectroscopical measurements of the metallicity. This strongly limits the size of the sample of objects one can use. DC99 selected 7 clusters, namely: NGC1851, NGC4590 (M68), NGC5272 (M3), NGC6171 (M107), NGC6362, NGC6981 (M72) and NGC7078 (M15). Data for other two clusters have been now added: IC4499 (Walker & Nemec 1996) and NGC5904 (M5) (Caputo et al. 1999). For IC4499, we adopt the iron abundance provided by Cohen et al. (1999), [Fe/H] = -1.46, whose metallicity scale is consistent with the one of Carretta & Gratton (1997, hereinafter CG97) adopted in DC99. In the case of M5, we have adopted the iron content listed by CG97.

It is worth noticing that for this cluster, in order to test the accuracy of our estimate, we have obtained the absolute bolometric ZAHB luminosity by using also the larger sample of variables investigated by Reid (1996). However, since Reid (1996) does not provide the blue amplitude (A_B) needed for estimating the effective temperature, we have derived A_B from the visual amplitude (A_V) , by using the relation:

$$A_B = 1.26 \cdot A_V + 0.04 \cdot [Fe/H] + 0.08$$
 1)

with a probable error p.e.=0.016 and a correlation coefficient r=0.997, obtained from a sample of 12 field variables observed by Liu & James (1990). It is interesting to notice that by using the two independent samples of variables, we have obtained the same value for $L_{2.87}^{ZAHB}$.

have obtained the same value for $L_{3.85}^{ZAHB}$.

Once obtained the value of $L_{3.85}^{ZAHB}$ for each cluster, the absolute visual magnitude of the ZAHB can be easily derived by using the relation:

$$M_V^{ZAHB} = -2.5 \cdot \log L_{3.85}^{ZAHB} - (BC_{3.85} - BC_{\odot}) + M_{V,\odot} 2$$

where $BC_{3.85}$ is the bolometric correction of a ZAHB star at $\log T_e=3.85$. By using the bolometric correction scale by Castelli, Gratton & Kurucz (1997a,b), we have estimated that:

$$(BC_{3.85} - BC_{\odot}) = 0.04[Fe/H] + 0.14$$

with a r.m.s. equal to 0.005 mag. For the absolute visual magnitude of the Sun, we adopt $M_{V,\odot}=4.82\pm0.02$ mag (Hayes 1985).

For all clusters in our sample, Table 1 reports the most relevant quantities: the name of the cluster, the adopted visual magnitude of the ZAHB, the iron abundance, the absolute bolometric luminosity of the ZAHB at $\log T_e = 3.85$, the absolute visual ZAHB magnitude, the distance modulus and the reddening. For the clusters with both fundamental and first overtone variables, the result listed has been obtained by averaging the values estimated from the RRab and RRc variables. The reddening values have been derived by comparing in the $(B-V)-A_B$ plane, the observational data for the RRab Lyrae stars with the empirical relation derived by Caputo & De Santis (1992).

It is worth investigating how much our predicted ZAHB absolute visual magnitudes are affected by systematic uncertainties in the main ingredients adopted in the method previously discussed. In DC99, we showed that when moving from the old pulsation relation, i.e. the relation connecting the period, stellar mass, luminosity, and effective temperature, provided by van Albada & Baker (1971) to the recent relation by Bono et al. (1997) the difference in the predicted M_V^{ZAHB} is quite small and roughly equal to 0.04 mag. A further pulsational input we are adopting is the position in the HR diagram of the fundamental instability strip. Recently, Caputo et al. (2000) have investigated the dependence of the First Overtone Blue Edge (FOBE) on the Helium abundance and the uncertainty on the treatment of superadiabatic convection. They have found that the location of the instability strip is marginally affected by mild He variations. Moreover, they also tested that current uncertainties in the calibration of the mixing length parameter cause a change (see discussion in Caputo et al. 2000) in the period at the FOBE at most of the order of $\Delta \log P = \pm 0.03$. This difference implies an uncertainty in the temperature of the FOBE of $\Delta \log T_e(FOBE) = \pm 0.009$. If we assume that this shift affects simultaneously the blue and the red edge of fundamental and first overtone pulsators then the maximum estimated error on the fundamental RR Lyrae masses is smaller than $0.005M_{\odot}$. The impact of this uncertainty on predicted M_V^{ZAHB} is negligible and roughly equal to 0.005 mag. As far as the uncertainty of current RR Lyrae temperature scale is concerned, DC99 emphasized that the probable error affecting the temperature estimate for each variable is equal to $\Delta \log T_e = \pm 0.003$. If we assume that the temperature determinations of the entire RR Lyrae sample in a cluster are systematically affected by this uncertainty, then the predicted ZAHB absolute visual magnitude is affected by an error of the order of 0.03 mag. As a whole, by accounting for all the previous error sources, we find that current M_V^{ZAHB} values could be affected, at most, by uncertainties

of the order of 0.05 mag. The values of M_V^{ZAHB} for all clusters, but NGC6171 (see below), are shown in Figure 1a. As far as it concerns the uncertainty on [Fe/H], we account for a realistic indetermination of about 0.15 dex (see Rutledge, Hesser & Stetson 1997). By using the data plotted in this Figure and listed in Table 1, we can now derive the $M_V - [Fe/H]$ relation by performing a best fit of the observational points. Since DC99 have shown that the value of $L_{3.85}^{ZAHB}$ for NGC6171 is affected by a large uncertainty, due to the poor quality of the available photometry, it has not been taken into account in order not to bias the solution.

By accounting for the uncertainty on both the absolute visual magnitude and the metallicity we derive the following relation:

$$M_V^{ZAHB} = (0.17 \pm 0.03) \cdot [Fe/H] + (0.87 \pm 0.04)$$
 4)

In view of the significant uncertainties still affecting the GCs metallicity scale, we have decided to perform the $M_V(ZAHB)-[Fe/H]$ calibration by using also the Zinn & West (1984) metallicity scale. The value of $L_{3.85}^{ZAHB}$ for each cluster in our sample has been recomputed by using the Zinn & West (1984)'s scale, and the final calibration is the following:

$$M_V^{ZAHB} = (0.17 \pm 0.03) \cdot [Fe/H] + (0.88 \pm 0.05)$$
 5)

Figure 2. Comparison in the $(\log P + 0.33 < V >) - \log T_e$ diagram between the RR_{ab} variables in different GCs and the prescriptions provided by the equation 5), when fixing the GC distance modulus to the value listed in Table 2, and adopting for the allowed minimum and maximum variable mass the values provided by the stellar evolutionary theory. Temperature scale is from De Santis (1996).

which is in fine agreement with the result based on the CG97 scale.

Since in the literature, different relations between the absolute magnitude of HB stars and the heavy elements abundance can be found, we have decided to compare the most recent ones with our solution. This has been done in fig. 1b. Since in DC99, we have already compared our determinations of $L_{3.85}^{ZAHB}$ with the most significant theoretical evaluations, now we limit the comparison to the empirical determinations of the $M_V(RR) - [Fe/H]$ relation. More in detail, we take into account the solutions given by Walker (1992), Sandage (1993), Clementini et al. (1995), Feast (1997), Gratton et al. (1997), Fernley et al. (1998), Groenewegen & Salaris (1999) and Caputo et al. (2000).

Since all these relations refer to the mean magnitude of the RR Lyrae, we have applied a correction to our solution in order to obtain the RR Lyrae mean magnitude from the ZAHB luminosity level:

$$\langle V_{RR} \rangle = V_{ZAHB} - 0.04 \cdot [Fe/H] - 0.15$$
 6)

provided by Cassisi & Salaris (1997, but see also Carney et al. 1992).

From data in fig. 1b), one can easily notice that this relations is in satisfactory agreement (within ≈ 0.07 mag) for [Fe/H] < -1.5 with the corresponding relation obtained

by Caputo et al. (2000) in the same metallicity range, when assuming a solar-scaled distribution for the heavy elements. However, we are not able to assess the existence of a change in the slope of the $M_V(RR)$ – [Fe/H] relation as disclosed by the quoted authors due to small size of the cluster sample, and in particular to the lack of GCs in the relevant metallicity range: -2.0 < [Fe/H] < -1.5. As a consequence, the two solutions differ also by about ≈ 0.10 mag at the upper metallicity limit ($[Fe/H] \approx -1$) we explore.

It seems also to exist a satisfactory agreement (at the level of less than 0.1 mag) with the relations given by Walker (1992), Groenewegen & Salaris (1999) and by Gratton et al. (1997). Therefore, present result provides further support to the "long" distance scale. On the contrary, an evident disagreement exists with the results based on the Baade-Wesselink method, as those provided by Clementini et al. (1995), Fernley et al. (1998) and Feast (1997).

A PULSATIONAL APPROACH TO THE GCS DISTANCE.

In Table 1, we have reported for each cluster the apparent distance modulus as obtained by using the V_{ZAHB} estimates following DC99, and the value of M_V^{ZAHB} obtained in the previous section. DC99 have already shown

Table 1.	The main	properties o	f the selected	sample of	globular	clusters.
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NGC	Name	V_{ZAHB}	$[\mathrm{Fe}/\mathrm{H}]$	$\log L_{3.85}^{ZAHB}$	M_V^{ZAHB}	$(m-M)_V$	E(B-V)
6171	M107	15.85 ± 0.10	-0.87	1.54 ± 0.04	0.874 ± 0.10	14.98 ± 0.02	0.38 ± 0.02
6362		15.33 ± 0.03	-0.96	1.615 ± 0.015	0.693 ± 0.03	14.64 ± 0.01	0.07 ± 0.013
1851		16.13 ± 0.025	-1.08	$1.645 {\pm} 0.01$	$0.618 {\pm} 0.025$	15.51 ± 0.01	$0.045 {\pm} 0.016$
5904	M5	15.20 ± 0.05	-1.11	1.60 ± 0.03	$0.733 {\pm} 0.08$	14.47 ± 0.03	0.06 ± 0.02
6981	M72	17.07 ± 0.03	-1.30	1.635 ± 0.01	$0.653 {\pm} 0.03$	$16.42 {\pm} 0.01$	$0.07 {\pm} 0.015$
5272	M3	15.73 ± 0.02	-1.34	1.645 ± 0.01	$0.628 {\pm} 0.02$	15.10 ± 0.01	0.012 ± 0.01
	IC4499	17.72 ± 0.05	-1.46	1.65 ± 0.03	0.619 ± 0.08	17.10 ± 0.05	$0.21 {\pm} 0.01$
4590	M68	15.70 ± 0.02	-1.99	1.70 ± 0.02	$0.506 {\pm} 0.02$	15.19 ± 0.03	0.043 ± 0.014
7078	M15	15.92 ± 0.05	-2.12	$1.68 {\pm} 0.02$	$0.556 {\pm} 0.05$	15.36 ± 0.03	0.074 ± 0.014

that the major source of uncertainty in the estimation of $L_{3.85}^{ZAHB}$ (and, in turn, of M_V^{ZAHB}) relies in the evaluation of the apparent magnitude of the ZAHB (see also the discussion in the previous section). However, it is worth emphasizing that, due to the approach adopted for deriving M_V^{ZAHB} , the measurement of the distance modulus $(m-M)_V = V_{ZAHB} - M_V^{ZAHB}$ is no more affected by any possible uncertainty in the choice of the ZAHB level.

This is a quite important point, since it means that the main source of uncertainty in the measurement of the distance modulus is related to the indetermination - usually very small - on $L_{3.85}^{ZAHB}$ due to the fit procedure in the period - effective temperature diagram between theory and observations. As a consequence, this method allows us to determine the distance modulus of clusters with a rich population of variables and spectroscopical measurements of their metallicity, with an uncertainty usually very small (≤ 0.03 mag).

In passing, we wish to notice that our distance modulus estimation for M3 appears in fine agreement with the distance recently derived by Bono et al. (2001) by adopting an independent method namely the K-band period-luminosity relation of RR Lyrae (15.03 \pm 0.07 mag).

In the following, we wish to outline a method useful for measuring the distance modulus of galactic GCs, based only on the pulsational properties of their RR Lyrae population. This method appears particularly attractive in case of GCs, showing a very blue HB (usually the ones with HB type larger than 0.8). In fact, in these clusters, it is no more possible to identify the lower envelope of the observed HB as the ZAHB locus. In fact, many (if not all) stars, within the RR Lyrae instability strip, are significantly evolved objects, crossing the strip at magnitudes brighter than the ZAHB level, during their evolution toward the Asymptotic Giant Branch

Besides, we notice that, whereas for determining the $L_{3.85}^{ZAHB}$ value for each cluster, one must rely on homogeneous photometry for both variable and non-variable HB stars (DC99), for obtaining the distance modulus by using the following approach, one needs to know only the mean magnitudes and the pulsational properties of RR Lyrae variables.

The suggested method works as follows: for each cluster variable we implement eq. 2) in DC99 with eq. 2) and 3) of the present paper, and the fundamental pulsational equation can be easily written in the following form:

$$\log P + 0.33 \cdot \langle V \rangle = 0.33 \cdot (m - M)_V - 0.33 \cdot \Delta BC +$$

$$-0.013 \cdot [Fe/H] - 0.582 \log M - 3.506 \log T_e + 13.171$$
 6)

where < V > is the mean visual magnitude, $(m-M)_V$ is the apparent distance modulus of the cluster, $\Delta BC = BC_{\log T_e} - BC_{3.85} = -5.252(\log T_e)^2 + 41.636\log T_e - 82.454$ i.e. the difference between the bolometric correction of a HB structure in the instability at $\log T_e = 3.85$ and the one with the same effective temperature of the variable (see DC99); the other quantities have their usual meaning. By using the same approach as in DC99, it is evident that once fixed the cluster metallicity and the allowed mass range for the RR Lyrae variables, one can use the $(\log P + 0.33 < V >) - \log T_e$ diagram for constraining the $(m-M)_V$ value.

Concerning the evaluation of the minimum and maximum mass for fundamental pulsators, we follow the approach suggested by DC99, which is based on the determination, for each metallicity, of the structures spending within the instability strip a significant amount ($\approx 20\%$) of their whole core He-burning phase. This approach relies on evolutionary lifetimes within the instability strip, and does not account for individual cluster HB morphology. However, synthetic HB experiments disclose that our approach safely estimates the mass range of variables in metal-poor clusters. On the other hand, in the case of intermediate metallicity clusters, affected by the 2nd parameter effect, the simulations show that our method safely estimates the minimum stellar mass which produces RR Lyrae stars, but slightly overestimate the maximum pulsator mass. Nevertheless, due to the dependence of $(\log P + 0.33 \cdot \langle V \rangle)$ on the variable mass and to the fit procedure between theory and observations, it results that an error on the upper mass limit of the order of $\approx 0.05 M_{\odot}$, causes a change in the estimation of the GC distance modulus of the order of ≈ 0.02 mag.

In order to show better how this method works, we have applied it to a selected sample of clusters. In particular, we have chosen the following GCs: M92, NGC6426, NGC5053, NGC5466, M55, M9 and M2. One has to notice that for the clusters M9, M55 and NGC 6426 only the RR_{ab} V amplitude are available. Since the adopted pulsational temperature scale (De Santis 1996) has been calibrated on blue amplitude, we used eq. 1) to obtain the blue amplitude for each variable in the sample.

In the case of the cluster NGC6426, we have not accounted for one (variable V16) out of the 8 variables investigated by Papadakis et al.(2000), as the classification of this variable is ambiguous, being suspected to be a c type variable. In the case of M9, we have omitted the variable V7

because its apparent magnitude is affected from an obscuring cloud to the southwest of the cluster.

We wish to notice that all these clusters are characterized by blue HB morphology. In Table 2, we list for each cluster in this sample, the reference for the RR Lyrae data, the adopted iron abundance as provided by CG97, the HB type defined as (B-R)/(B+V+R)—where B,V and R are the numbers of stars hotter than the RR Lyrae instability strip, of RR variables, and of stars cooler than the instability stripprovided by Harris (1996), the estimated mass range for fundamental pulsators and the distance modulus obtained by using the previously described approach and the related uncertainty.

In Figure 2, for each cluster we show the comparison between theory and observations in the (log P+0.33 < V >) - log T_e diagram. One can notice that, once fixed the minimum and maximum allowed mass for fundamental pulsators, the observational distribution is well matched only when fixing the GC distance modulus to the value listed in Table 2.

The estimate of the M92 distance modulus appears in satisfactory agreement with the results provided by Pont et al. (1998), Carretta et al. (2000) and Vandenberg (2000); and within the listed uncertainties also with the estimation given by Reid & Gizis (1998). As far as the GCs in common with the work of Caputo et al. (2000), we verify a general agreement within 0.1 mag. However, for two clusters, namely NGC5053 and M2, it exists a large discrepancy of 0.18 and 0.16 mag respectively.

We wish to notice that this method, when applied to GCs whose RR Lyrae instability strip is well populated, as for instance the clusters listed in Table 1, provides distance determinations fully consistent with the ones obtained by using the approach adopted in the previous section for deriving $L_{3.85}^{ZAHB}$, and in turn, M_V^{ZAHB} .

4 CONCLUSIONS.

In present work, by adopting the same approach discussed in DC99, we increase the sample of galactic GCs for which we derive the absolute bolometric magnitude of the ZAHB ($L_{3.85}^{ZAHB}$) at the level of the RR Lyrae instability strip. As already shown in DC99, this occurrence is quite important in order to test the accuracy and reliability of the current theoretical predictions on this quantity. In addition, we now use the obtained values for $L_{3.85}^{ZAHB}$ in order to estimate, for each cluster in our sample, the absolute visual magnitude of the ZAHB. This allows us to investigate the dependence of this parameter on the cluster metallicity, by deriving a $M_V(ZAHB) - [Fe/H]$ relation. Due to the limited number of GCs in our sample, we can not assess, as made by Caputo et al. (2000), the existence of a non-linear dependence of M_V^{ZAHB} on the cluster metallicity.

The comparison of our $M_V(ZAHB) - [Fe/H]$ relation - after rescaling it for the luminosity difference between the ZAHB and the mean RR Lyrae magnitude -, with the most recent empirical determinations of $M_V(RR) - [Fe/H]$ relation, shows that our result is in satisfactory agreement with almost all measurements supporting the "long" distance scale as the ones provided by Gratton et al. (1997), Groenewegen & Salaris (1999) and Walker (1992). Present result is also in good agreement with the Caputo et al. (2000) determination for metallicity lower than -1.5 dex.

For larger metallicity, since the relation by Caputo et al. (2000) has a stronger dependence on metallicity than our one, it exists a significant discrepancy, increasing with the metallicity, also of the order of 0.1 mag. A clear discrepancy appears between our distance scale and all the other ones supporting the "short" distance scale as the ones based on Baade-Wesselink method (Clementini et al. 1995, Fernley et al. 1998).

We present a method for determining the distance to galactic GCs, based only on the pulsational properties of RR Lyrae stars. This method consists in comparing observations and expectations provided by updated pulsational and evolutionary models in the (log $P+0.33 \cdot < V >$) – log T_e diagram.

From a theoretical point of view, our approach relies on predictions like the dependence of the fundamental pulsational equation on the evolutionary properties of the variable, and the allowed mass range for fundamental pulsators, therefore the accuracy of the obtained results depends on the reliability of the adopted theoretical framework. However, since current theoretical predictions on both the fundamental pulsation equation and the ranking of HB stellar mass as a function of the effective temperature, appear quite robust, we think that this method can provide accurate distance determinations. Nevertheless, it does also need the use of a temperature scale for RR Lyrae stars, whose reliability is a long-standing problem (see, for instance, Catelan (1998) and Carretta, Gratton & Clementini (2000)). However, in DC99 (see also De Santis 2001) we have carefully checked the accuracy of the temperature scale provided by De Santis (1996) and adopted in the present investigation. So, we are confident that our distance modulus determinations are not significantly affected by the residual uncertainty affecting the RR Lyrae temperature scale.

The reliability of the suggested method has been shown by deriving the distance moduli of a selected sample of galactic GCs, for which the determination of the distance through the usual HB fitting has been always a risky procedure due to the morphology of their HB. The derived distances appear, within the uncertainty, in satisfactory agreement with the values listed in the more recent literature.

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Table 2. The main properties of the GC sample to which has been applied our method for measuring the distance modulus.

NGC	Name	Source	[Fe/H]	HB Type	M_{RR}/M_{\odot}	$(m-M)_V$
6341	M92	Carney et al. (1992)	-2.15	0.90	0.70 - 0.80	14.71 ± 0.05
6426		Papadakis et al. (2000)	-2.07	0.58	0.70 - 0.80	17.80 ± 0.07
5053		Nemec et al. (1995)	-2.10	0.52	0.70 - 0.80	16.13 ± 0.05
5466		Corwin et al. (1999)	-2.03	0.58	0.70 - 0.80	16.05 ± 0.03
6809	M55	Olech et al. (1999)	-1.65	0.87	0.68 - 0.74	13.90 ± 0.05
6333	M9	Clement & Shelton (1999)	-1.57	0.87	0.67 - 0.73	15.75 ± 0.05
7089	M2	Lee & Carney (1999)	-1.34	0.96	0.64 - 0.70	15.61 ± 0.05

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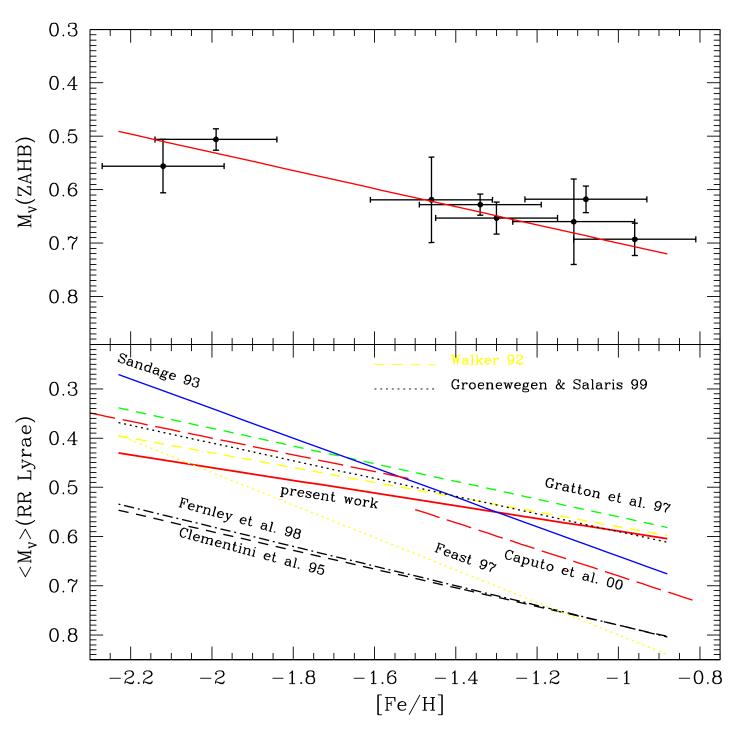


Figure 1

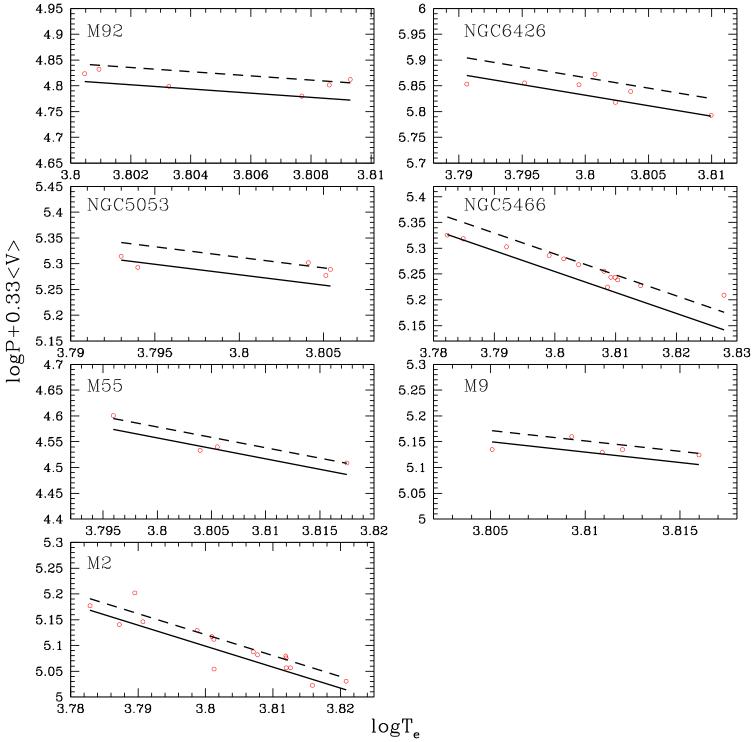


Figure 2)